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*Invariance of Domain for $\mathcal{A}_G(S_+)-$Operators*

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Let $X$ be a real reflexive Banach space and $X^*$ its dual space. Let $T : X \supset D(T) \to 2^{X^*}$ be an operator of class $\mathcal{A}_G(S_+)$, where $G \subset X$ is open. An invariance of domain result for $T$ will be discussed. This result extends a similar result of Park for single-valued operators of type $(S_+)$. The Skrypnik's topological degree theory is used, utilizing approximating schemes of operators of class $\mathcal{A}_G(S_+)$, along with the methodology of a recent invariance of domain result by Kartsatos and the author.

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*Optimal treatment and vaccination in an SIR Epidemiological Model with Time-varying Population*

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Coauthors: Urszula Ledzewicz, Heinz M.Schaettler

The problem of optimizing treatment and vaccination rate in an SIR model with time-varying population is formulated as an optimal control problem where the intervention strategies used to limit the spread of the disease. It is shown that singular controls play an important role in the structure of optimal vaccination schedules whereas optimal treatment schedules follow a bang-bang structure of on-off strategies.

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*On a Krylov Subspace Approximation to the Matrix Exponential Operator for Sparse Matrices*

**Dashti Ahmed Ali**
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The matrix exponential operation $e^A\varphi$, where A is a square matrix and $\varphi$ is an initial vector, commonly arises in the applications of various scientific fields due to the fact that it can provide the solution of the systems of linear differential equations emerging in the mathematical modelling of scientific problems. When the matrix A is large, Krylov subspace methods are in high priority to compute $e^A\varphi$ since these methods are more time efficient than the other basic methods in literature. This article aims to investigate and analyse a Krylov subspace method and the scaling and squaring method in terms of accuracy and efficiency. For this purpose, we have carried out a number of numerical experiments in MATLAB programming framework. In addition, we particularly focused on sparse type of matrices since they regularly appear in real world applications.

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*Stochastic Differential Equation Models for the Wear of Coins in Circulation*

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Two hypotheses for the wear of coins in circulation are mathematically examined. In the first hypothesis, the wear rate is assumed to be the same at all points on the coin surface. In the second hypothesis, high points on the coin surface are assumed to wear at a greater rate than low points. For each hypothesis, deterministic and stochastic differential equation (SDE) models for the dynamical wear of coins in circulation are derived from physical principles. The stochastic and deterministic

models are solved and compared. The models agree with and explain the actual weight-loss behavior of coins in circulation. As an application of the models, a quantitative parameter of coin condition suggested by the models is described and studied.

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*The Impact of Insect Aggregation and Dispersal on Disease Outbreaks in Insect-plant-virus Models*

**Linda J.S. Allen**
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While feeding on host plants, viruliferous insects serve as vectors for viruses. Successful viral transmission depends on vector behavior. Two behaviors that impact viral transmission are vector aggregation and dispersal. Vector aggregation may be due to chemical or visual cues or feeding preferences. Vector dispersal can result in widespread disease outbreaks among susceptible host plants. These two behaviors are investigated in insect-plant-virus models. Deterministic and stochastic models are formulated to account for stages of infection, vector aggregation and local dispersal between adjacent crops. First, models for a single crop are studied with aggregation included implicitly through the acquisition and inoculation rates. Second, models with aggregation and dispersal of vectors are studied when one field contains a disease-sensitive crop and another a disease-resistant crop. Analytical expressions are computed for the basic reproduction number in the deterministic models and for the probability of disease extinction in the stochastic models. These two expressions provide useful measures to assess effects of aggregation and dispersal on the rate of disease spread within and between crops and the potential for an outbreak. The modeling framework is based on cassava mosaic virus that causes significant damage in cassava crops in Africa.

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*Investigation of the Use of Razumikhin Functions to Analyze the Stability of NFDE Solutions to the PEEC Model*

**H. Michael Allison**
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A. Ruehli's Partial Element Equivalent Circuit (PEEC) Model is used to solve Electrical Field Integral Equations (EFIE) for conductors in 3-dimensional space such as traces in a circuit board. For frequencies with wavelengths comparable to the length of circuit board traces, time delays must be taken into account. The first harmonic of a 10GHz square wave is 3cm so this can happen with current high-speed digital circuits. Time delays result in NFDEs in the solutions to the PEEC model.

This paper investigates the use of Razumikhin functions to analyze the stability of solutions to the PEEC Model with delay, rPEEC, in contrast to the known Lyapunov-Krasovskii functional methods.

[1] A. E. Ruehli: Equivalent Circuit Models for Three-Dimensional Multiconductor Systems, IEEE Transactions on Microwave Theory and Techniques, Vol. MTT-22, No. 3, pp. 216-221, Mar. 1974.

[2] Antonini, G. Pepe, P. (2009). Input-to-State Stability Analysis of Partial-Element Equivalent Circuit Models. IEEE Transactions on Circuits and Systems I: Regular Papers, vol. 56, No. 3 673-684.

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*Ergodic Convergence of the Double Backward Method for Monotone Operators*

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The double backward method for finding zeros of the sum of two maximal monotone operators is investigated. This method was initially introduced by Passty in 1979 who used an equal index to the resolvents of both operators. In this paper, we use distinct indices in order to see different roles played by the two operators in the double backward method. Under certain conditions on the indices, we prove the ergodic convergence of our method.

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*High-order Finite Difference Technique for Delay Pseudo-parabolic Equations*
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One dimensional initial-boundary delay pseudo-parabolic problem is being considered.To solve this problem numerically, we construct higher order difference method for approximation to the considered problem and obtain the error estimate for its solution. Based on the method of energy estimate the fully discrete scheme is shown to be convergent of order four in space and of order two in time. Numerical example is presented.

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*Pursuit in an Inverse Square Law Medium*
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We consider a two-plow pursuit problem in which each plower's velocity is inversely proportional to the square of the material depth, such as accumulating snow. We build closed-form solutions to determine conditions such that the trailing plow will reach the leading plow, or grind to a halt short of a catch. The solution involves an imaginary shift alteration of a wavefront.

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*Analysis and Simulation of a Coupled Nonlinear Fluid-structure Interaction Mathematical Model with Applications to Aneurysms*
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Coauthors: JengEng Lin, and Padmanabhan Seshaiyer

In this work, we will present a mathematical model that describes a coupled fluid-structure interaction between blood flow and the arterial wall with applications to intracranial saccular aneurysm. The governing differential equations include a nonlinear power-law fluid equations coupled with nonlinear elasticity equations in conjunction with blood pressure that is modeled via a Fourier series. The thrust of this work will involve the analysis and simulation of the associated mathematical model using classical differential equation techniques. An exact solution to the governing nonlinear differential equation is derived for a special class of problems that is biologically tractable.

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*Crystalline and Complex Solutions to the Vector Allen-Cahn Equation*
**Peter Bates**
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We study the problem of finding bounded entire solutions $u : \mathbb{R}^n \to \mathbb{R}^m$ to the vector Allen-Cahn equation $\Delta u - W_u(u) = 0$, for all $x \in \mathbb{R}^n$, where $W : \mathbb{R}^m \to \mathbb{R}$ is a smooth, symmetric, nonnegative multi-well potential that vanishes at $N$ distinct points $a_1, a_2, , a_N \in \mathbb{R}^m$, and $W_u = \nabla W$. Even in the case $m = 1$ and $N = 2$, the problem of identifying all solutions is very difficult (see the work stemming from De Giorgi's conjecture, and relatively recent work by Dancer, Malchiodi, Del Pino - Kowalczyk - Wei, for instance). Here we present a large family of symmetric solutions in the general case with W described above. This is joint work with Giorgio Fusco and Panayotis Smyrnelis.

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*A Method of Directly Defining the Inverse Mapping for Solutions of Coupled Systems of Nonlinear Differential Equations*
**Mathew Baxter**

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Coauthors:Mangalagama Dewasurendra, and Kuppalapalle Vajravelu

The Homotopy Analysis Method (HAM) uses the methods of perturbation to find analytical approximate solutions to nonlinear differential equations where only numerical solutions exist. Recently, Liao introduced a new method that is an extension to HAM. In this talk, we extend this idea to nonlinear systems. We study the system of nonlinear differential equations that governs nonlinear convective heat transfer at a porous flat plate, and find functions that approximate the solutions by extending Liao's Method of Directly Defining the Inverse Mapping (MDDiM).

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*Stability and Persistence in Multi-Epitope HIV-Immune Response Network Models*

**Cameron Browne**
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The CTL (Cytotoxic T Lymphocyte) immune response plays a large role in controlling HIV infection. CTL immune effectors recognize epitopes (viral proteins) presented on the surface of infected cells to mediate their killing. The immune system has an extensive repertoire of CTLs, however HIV can evolve resistance to attack at different epitopes. The ensuing arms race creates an evolving network of viral strains and CTL populations with variable levels of epitope resistance. Motivated by this, we formulate a general ODE model of multi-epitope virus-immune response dynamics. Some special cases for the HIV/CTL interaction network are considered, such as the case of a nested network and the general two-epitope case. We characterize the uniform persistence of viral strains and immune response variants, stability of equilibria and prove global properties of solutions via Lyapunov functions. The results are interpreted in the context of within-host HIV/CTL evolution and numerical simulations are provided. To conclude, we discuss extensions of the model to a PDE system which incorporates cell-infection age structure.

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*Controllability of Fractional Neutral Impulsive Integro-differential Equations in Banach spaces*

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Coauthor: Ravi Chandran

In this paper, we prove the controllability for impulsive neutral fractional integrodifferential equations in Banach spaces using Banach fixed point theorem, semigroup theory and fractional calculus. We present the controllability result by introducing a class of distributed controls which are highly useful for the computational purpose also. The controversy on the solution operator is discussed here and we emphasize that we use the generalized Caputo derivative with the lower bound at zero for the system considered. An example is given to illustrate the abstract results.

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*The Mathematics of Dispersion for Metamaterials*

**Yue Chen**
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Metamaterials are a new form of structured materials designed to have electromagnetic properties not generally found in nature. In this talk, I will introduce a rigorous mathematical framework for controlling localized resonances and predicting exotic behavior inside optical metamaterials. The theory is multiscale in nature and provides a rational basis for designing microstructure using multiphase nonmagnetic materials to create backward wave behavior across prescribed frequency ranges.

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*Optimal Control of the Coefficients in Second Order Parabolic Free Boundary Problems*

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Coauthors: Ugur G. Abdulla and Jonathan Goldfarb

Inverse Stefan type free boundary problem for the second order parabolic PDEs with unknown coefficient is considered. Optimal control framework is employed where coefficient of the PDE and free boundary are components of the control vector. We prove the Frechet differentiability in Besov spaces and derive the formula for the Frechet gradient under the minimal regularity assumptions on the data. Necessary condition for the optimality is formulated and projective gradient method in Besov-Hilbert spaces framework is implemented. Numerical results of model examples are presented.

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*An Analysis of the Caputo Fractional Derivative Using the Laplace Transform*

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Coauthor: Richard Noren

The Caputo Fractional Derivative equation has been presented in other papers before and has been treated with numerical solutions that are dependent in both time and space. Equations involving a fractional derivative are known to have many applications tied to diffusion and dispersion, and the Caputo Fractional Derivative equation is no different.

Our goal is to remove the dependence on a time-based scheme and simply have approximations in space, and this is most easily accomplished by the use of the Laplace Transform. We use the Laplace Transform on the integro-differential equation to remove the time-derivative presented in the original. This preserves a convolution integral with our Laplacian in space, with the time derivative removed. After which, some mild analysis is included on the existence and uniqueness of a solution for this new equation, and a general solution is obtained by use of further transform methods to solve. Finally, a simple example is considered where we may apply a numerical approximation method on the Laplacian in space, which leaves us without any dependence on our time variable. Currently we are seeking other avenues on our numerical treatment, which will be addressed. I will conclude with a comparison to one of the methods presented in a recent paper, and address the strengths and weaknesses of both.

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*Generalized Peano Existence and Resulting Monotone Method for*
*Nonlinear Riemann-Liouville Fractional Differential Equations*

**Zachary Denton**
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A generalized Peano's existence theorem is developed for Riemann-Liouville fractional differential equations when the forcing function is not bounded, but is instead $C^p$ continuous with a weighted bounded condition. This result leads to another existence result via upper and lower solutions, along with a comparison theorem for upper and lower solutions. These results together yield a construction of the Monotone Method iterative technique for approximating nonlinear RL FDEs where we relax the conditions on $f$ to be $C^p$ continuous.

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*On the Semigroup Generator for the Total Linearization of a Hydro-Elasticity Model*

**Steven Derochers**
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Coauthors: Lorena Bociu and Daniel Toundykov

In this talk, we consider a fluid structure system involving coupled fluid flow. In the examination of the well-posedness of this system, we utilize a nonstandard means to eliminate the pressure term. This process involves specific extensions into the solid domain. Unlike the standard model, the well-posedness of this system depends on the fluid's viscosity and new terms on the interface which involve, among other things, the curvature of the boundary. Furthermore we implement a finite element scheme for approximating solutions of a prototype system to numerically investigate the dependence of the discretized model on the "new" terms not present in the standard system.

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*Lyapunov-type Inequalities with Fractional Integral Boundary Conditions*

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We discuss boundary value problems for Riemann-Liouville fractional differential equations with certain fractional integral boundary conditions. Such boundary conditions are different from the widely considered pointwise conditions in the sense that they allow solutions to have singularities, and different from other conditions given by integrals with a singular kernel since they arise from well-defined initial value problems. We derive Lyapunov-type inequalities for linear fractional differential equations and apply them to establish nonexistence, uniqueness, and existence-uniqueness of solutions for certain linear fractional boundary value problems. Parallel results are also obtained for sequential fractional differential equations. An example is given to show how computer programs and numerical algorithms can be used to verify the conditions and to apply the results.

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*Modeling the Mechanisms by Which HIV-associated Immunosuppression Influences HPV Persistence at the Oral Mucosa*

**Samantha Erwin**
*Virginia Tech*

Human immunodeficiency virus (HIV)-infected patients are at an increased risk of co-infection with human papilloma virus (HPV), and subsequent malignancies such as oral cancer. To determine the role of HIV-associated immune suppression on HPV persistence and pathogenesis, we developed a mathematical model of HIV/HPV co-infection and used it to investigate the mechanisms underlying the modulation of HPV infection and oral cancer by HIV. Our model captures known immunological and molecular features and enhanced HPV infection due to HIV. We used the model to determine HPV prognosis in the presence of HIV infection, and identified conditions under which HIV infection alters HPV persistence in the oral mucosal system. The model predicts that conditions leading to HPV persistence during HIV/HPV co-infection are the permissive immune environment created by HIV and molecular interactions between the two viruses. The model also determines when HPV infection continues to persist in the short run in a co-infected patient undergoing antiretroviral therapy. Lastly, the model predicts that under efficacious antiretroviral treatment HPV infections will decrease in the long run due to the restoration of CD4+ T cell levels.

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*Free Boundary Problems Arising in Biology*

**Avner Friedman**
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In a free boundary problem one seeks to solve a system of PDEs in a domain G whose boundary, or a part of it, is unknown, and to also determine the free boundary. Classical free boundary problems include contact problems in elasticity, melting of ice, propagation of jets, and cavitational flows. In recent years new free boundary problems arose in the context of biological or biomedical processes. Examples include the healing of a wound, the growth of a tumor, the formation of a plaque in the artery (atherosclerosis) which leads to a heart attack or a stroke, the development of granulomas in tuberculosis, abdominal aorta aneurysm, and biofilms. In this talk I will briefly describe these biological problems, introduce their mathematical models, and display simulations of the models and their biological significance. Finally, I will review rigorous mathematical results for the PDE models, and state open problems.

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*Vibrational Energies of the Hydrogen Bonds of $H_3O_2^-$ and $H_5O_2^+$*

**Stephanie Gamble**
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We approximate the vibrational energies of the symmetric and asymmetric stretches of the hydrogen bonds of the molecules $H_3O_2^-$ and $H_5O_2^+$ by applying an improvement to the standard time-independent Born-Oppenheimer approximation. These two molecules are symmetric around a central hydrogen which participates in hydrogen bonding. Unlike the standard Born-Oppenheimer approximation, this approximation appropriately scales the hydrogen nuclei differently than the heavier oxygen nuclei. This results in significantly more accurate approximations for the stretching vibrational energies, which we compare to experimental measurements.

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*Frechet Differentiability in Besov Spaces in the Optimal Control of Parabolic Free Boundary Problems*

**Jonathan Goldfarb**
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Coauthor: Ugur Abdulla

We consider the inverse Stefan type free boundary problem, where information on the boundary heat flux and density of the sources are missing and must be found along with the temperature and the free boundary. We pursue optimal control framework where boundary heat flux, density of sources, and free boundary are components of the control vector. We prove the Frechet differentiability in Besov spaces, and derive the formula for the Frechet differential under minimal regularity assumptions on the data. The result implies a necessary condition for optimal control and opens the way to the application of projective gradient methods in Besov spaces for the numerical solution of the inverse Stefan problem.

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*Applying the Fractional Diffusion Equation to Financial Forecasting*

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In financial mathematics, capital asset pricing is always considered to be a critical problem which closely related to many areas of financial markets. Since published, the Black-Scholes Model becomes to one of the most effective tool to explain and forecasting financial derivatives. Along with the development of research, people realize that classical model cannot perfectly describe the nature of financial markets and forecast it. Fractal Market Hypothesis is one of the approaches to generalize the classical hypothesis by using the fractal theory. In this presentation, we are going to derive and generalize the fractional diffusion equation and find its Green's Function solution. Then we will discuss its application in finance and economics.

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*On Self-similar Solution to Navier-Stokes Equations in Marcinkiewicz Spaces*

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Some new local energy bounds are obtained for the time-dependent Navier-Stokes equations which imply the regularity condition $L_t^\infty(X)$, where $X$ is a non-endpoint borderline Lorentz space $X = L_x^{3,q}, q \neq \infty$. The analysis also allows us to rule out the existence of Leray's backward self-similar solutions to the Navier-Stokes equations with profiles in $L^{12/5}(\mathbb{R}^3)$ or in the Marcinkiewicz space $L^{q,\infty}(\mathbb{R}^3)$ for any $q \in (12/5, 6)$. This talk is based on joint work with Nguyen Cong Phuc.

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*Heterogeneous Virus Strategies Promotes Coexistence in Virus-Microbe Systems*

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Coauthor: Joshua Weitz

Viruses of microbes, including bacterial viruses (phage), archaeal viruses, and eukaryotic viruses, can influence the fate of individual microbes and entire populations. Here, we model distinct modes of virus-host interactions and study their impact on the abundance and diversity of both viruses and their microbial hosts. We consider two distinct viral populations infecting the same microbial population via two different strategies: lytic and chronic. A lytic strategy corresponds to viruses that exclusively infect and lyse their hosts to release new virions. A chronic strategy corresponds to viruses that infect hosts and then continually release new viruses via a budding process without cell lysis. The chronic virus can also be passed on to daughter cells during cell division. The long-term association of virus and microbe in the chronic mode drives differences in selective pressures with respect to the lytic mode. We utilize invasion analysis of the corresponding nonlinear differential equation model to study the ecology and evolution of heterogenous viral strategies. We first investigate stability of equilibria, and characterize oscillatory and bistable dynamics in some parameter regions. Then, we derive fitness quantities for both virus types and investigate conditions for competitive exclusion and coexistence. In so doing we find unexpected results, including a regime in which the chronic virus requires the lytic virus for survival and invasion. In closing, we discuss generalizations of the model and the long-term evolution of strategies across a spectrum of lytic to chronic modes.

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*Viscoelastic Wave Equations with Supercritical Nonlinearities*

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Coauthors: M. A. Rammaha, S. Sakuntasathien, E. S. Titi, and D. Toundykov

Viscoelastic materials demonstrate properties between those of elastic materials and viscous fluid. In the 19th century, Boltzmann realized that the behavior of these materials should be modeled through constitutive relations that involve long but fading memory, and in particular, he initiated the classical linear theory of viscoelasticity. In this talk, I will discuss a hyperbolic PDE, modeling wave propagation in viscoelastic media, under the influence of a linear memory term of Boltzmann type, and a nonlinear frictional damping, as well as an energy-amplifying supercritical nonlinear source. Our main interests lie in the well-posedness of weak solutions, interactions between sources and dissipation, and singularity formulation.

---

*Pseudospectral Methods in Optimal Control*

**William Hager**
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In an optimal control problem, the performance of a dynamical system is optimized with respect to parameters that enter into the dynamics. Examples range from a satellite or a rocket or an air plane where the controls might be fuel burn rate or the orientation of control surfaces, to the human body or a biological population where the controls could be dosage rates of medications. In these problems, the state of the system is described by a differential equation, and the goal is find controls that optimize the system performance subject to constraints on either the state or the controls.

Problems of this nature are usually too complex to solve analytically. Computationally, we need to replace the continuous infinite dimensional problem by a finite dimensional discrete problem. The talk will survey classical discretization techniques based on a Runge-Kutta approximation to the differential equations (an h-method) and then introduce recent approximations based on collocation at the roots of orthogonal polynomials (a p-method). The best approximations are often achieved using an hp-framework that combines the best features of both approaches.

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*Backward Stochastic Differential Equations and Applications*

**Imen Hassari**
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We are concerned with the problem of existence and uniqueness of a solution for the backward stochastic differential equations (BSDEs for short) with two continuous reflecting barriers. Roughly speaking, a solution for this equation is a quadruple of $\mathcal{F}_t$-adapted stochastic processes $(Y, Z, K^+, K^-)$ such that for any $t \in [0, T]$:

$$\begin{cases} Y_t = \xi + \int_t^T f(s, Y_s, Z_s)ds + (K_T^+ - K_t^+) - (K_T^- - K_t^-) - \int_t^T Z_s dB_s; \\ L_t \leq Y_t \leq U_t \textit{ and } \int_0^T (Y_s - L_s)dK_s^+ = \int_0^T (U_s - Y_s)dK_s^- = 0, \ \mathbb{P} - a.s. \end{cases}$$

where $B$ is a Brownian motion, $\xi$, $f$, $L$ and $U$ are the given data of the problem.

Recently, T.Klimsiak considered doubly reflected BSDEs with data in $L^p$ spaces, $p \in [1, 2]$ and with two irregular barriers $L$ and $U$ satisfying the generalized Mokobodzki's condition.
In this work, we consider that the data are $\mathbb{L}^p$-integrable with $p = 1$. Our generator satisfy the Lipschitz property, we make a growth condition of order 1 on $f$ with respect to $y$ and we assume that the generator has a growth condition of order $\alpha$, $\alpha \in [0, 1[$, with respect to $y$ and $z$. Under the above conditions on data we show existence and uniqueness of a solution which belongs to class $\mathbb{D}$.
The main result generalizes the result of Hamadne and Hassani to $\mathbb{L}^1$-data and in some direction the results of Klimsiak, where $\mathbb{L}^1$-data are considered but with barriers satisfying Mokobodzki's condition, which says that there exists a difference of two supermartingales cdlg uniformly integrable between the two barriers. The main contribution is the withdrawal of the Mokobodzki's condition, which is difficult to obtain or to verify in practice.
To illustrate the importance utility of this new result, we will present some applications concerning differential games and real options in finance.

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*Sharp Condition on Global Well-posedness for Nonlinear Schroedinger Equations with Rotation*

**Yi Hu**
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Coauthors: Nyla Basharat and Shijun Zheng

In this talk, we consider the nonlinear Schroedinger equation with rotation (also called Gross-Pitaevskii equations) in two and three dimensions, and give sharp conditions on the global well-posedness and blowup in the mass-critical case. Specifically, this condition is related to the ground state solution. We will also mention the blow up rate if time permits. This is a joint work with Nyla Basharat and Shijun Zheng.

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*Volterra Equations and Asymptotic Periodicity*

**Muhammad N. Islam**
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Existence of asymptotically periodic solutions of a nonlinear Volterra integral equation is studied in this paper. In the process, the existence of periodic solutions of an associated nonlinear integral equation with infinite delay is obtained. Schauder's fixed point theorem has been used in the analysis.

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*Discrete and Continuous Operational Calculus in Real Time Stochastic Games*

**Kenneth Ibe Iwezulu**
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We consider a class of antagonistic stochastic games in real time between two players A and B formalized by two marked point processes. The players attack each other at random times with random impacts. Either player can sustain casualties up to a fixed threshold. A player is defeated when its underlying threshold is crossed. Upon that time (referred to as the first passage time), the game is over.

In this paper, we introduce a joint functional of the first passage, along with the status of each player upon this time, meaning the cumulative magnitude of casualties to each player upon the end of the game, obtained in an analytically tractable form. We then use discrete and continuous operational calculus for the transform inversion. We demonstrate that in a special case that the discrete operational calculus is more efficient allowing us to avoid numerical inversion. It leads to totally explicit formulas for the joint distribution of associated random variables (first passage time and the status of cumulative casualties to the players upon the end of the game).

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*Invariant Manifolds of the Solitary Waves of the Supercritical gKDV equation*

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Coauthors: Zhiwu Lin and Chongchun Zeng

It is well-known that the solitary waves of the Supercritical gKDV equation is linearly unstable. Using the Hamiltonian structure of the linearized gKDV equation, we linearly decompose the energy space into stable space, unstable space, center space. Based on this linear decomposition, stable manifolds, unstable manifolds and center manifolds are constructed. The main difficulty of our problem is the loss of derivative in the nonlinearity and low regularity of the center space. To overcome these difficulties, we write the gKDV equation as a geometric equation over bundles and use certain space-time norm to gain regularity.

---

*Two Equivalent Lyapunov-Krasovskii Descriptions of Output Stability for Delay Systems*

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Coauthor: Yuan Wang

In current work we present two Lyapunov-Krasovskii characterizations for output stability of systems with delay affected by disturbances given as follows:

$$\begin{aligned} \dot{x}(t) &= f(x_t, d(t)), \\ x(s) &= \xi(s), \ \ -\theta \le s \le 0 \\ y(t) &= h(x(t)), \end{aligned}$$

where $x_t : [-\theta, 0] \to \mathbb{R}^n$ is given by $x_t(s) = x(t+s)$, $\theta > 0$ is a constant and the measurable, essentially bounded function $d$ represents a disturbance to the system. (Here basic regularity conditions are assumed to guarantee the basic properties of the system.) We obtain two Lyapunov characterizations of uniform output stability, by using two Lyapunov-Krasovskii functionals with different decay estimates. One of them is tailor-made for delay systems, whereas the other is motivated by the delay free case, which provides more flexibility in applications.

In the special case when there is no disturbance function $d$ and the output function $h$ being the identity, current result reduces to Lyapunov characterizations of global asymptotic stability in classical-system sense.

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*On Energy-stable Schemes for Complex-fluid Hydrodynamic Equations*

**Shane Kepley**
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Coauthors: William D. Kalies and J.D. Mireles-James

Complex fluids are fluids whose micro-structure have impact on the fluid macroscopic properties, which include complex fluid mixtures of different types. In this talk, I will first present a systematic development of a general hydrodynamic model for complex fluid system using the generalized Onsager relation. Then, a semi-discrete scheme to solve this general model, which satisfies the discrete energy dissipation law, will be presented. Specific tricks on linearizing and decoupling the schemes will be presented for particular reduced models. In the end, several 3D simulations will be shown to illustrate the effectiveness of our schemes.

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*Perturbation Formulas for Nonlinear Polyharmonic Equation with Periodic Potentials when $2l > n$*

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In this talk, we investigate the perturbation formulas for Nonlinear Polyharmonic Equation with Periodic Potentials when 2l ¿ n. In the first part of this study, we consider the perturbation formulas for Linear Schrödinger operator with periodic potential. In the second part, we use the results of the perturbation formulas of the linear equation to find a stationary solution and its corresponding value for the nonlinear. Here, we need a several methods, such as perturbation theory, spectral theory and successive method.

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*Eigenvalue Asymptotics for Zakharov-Shabat Systems with Long-range Potentials.*

**Martin Klaus**
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The Zakharov-Shabat system is a non-selfadjoint system of coupled differential equations that arises in the study of optical pulses by the inverse scattering technique. The underlying differential operator typically exhibits a nonreal spectral part consisting of discrete eigenvalues. Of particular interest are the purely imaginary eigenvalues. Under the usual conditions on the potential this number is finite. The main topic of this talk will be operators with an infinite number of imaginary eigenvalues which accumulate at the origin. We will consider the asymptotics of the number of imaginary eigenvalues that lie above $s$ ($s > 0$) as $s \to 0$. In the process, we will discuss the differences and similarities between this problem and an analogous one for Schrodinger operators.

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*Narrow-Stencil Finite Difference Methods for Approximating Viscosity Solutions of Hamilton-Jacobi-Bellman Equations*

**Tom Lewis**
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We introduce a new narrow-stencil finite difference (FD) method for approximating the viscosity solution of Hamilton-Jacobi-Bellman equations, and, more generally, second order fully nonlinear elliptic partial differential equations. The new FD method naturally extends the Lax-Friedrich's method for first order problems to second order problems by including a term called a numerical moment that acts as a stabilization term. The numerical moment uses multiple gradient and Hessian approximations to resolve the potential low-regularity of viscosity solutions, and it will be the key to overcoming the lack of monotonicity of the scheme while preserving consistency, admissibility, stability, and convergence.

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*A Posteriori Error Estimates Applied to Unconditionally Stable IPDG Methods for the Helmholtz Equation*

**Cody Lorton**
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Unconditionally stable interior penalty discontinuous Galerkin (IPDG) methods for the Helmholtz equation were developed and analyzed by X. Feng and H. Wu (2009, 2014). In this talk, we discuss the performance of residual-based a posteriori error estimates applied to this class of IPDG methods. Numerical experiments are included to demonstrate key features.

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*Positive Symmetric Solutions of a Second Order Boundary Value Problem*

**Jeffrey W. Lyons**
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Coauthors: Daniel Houston and Jeffrey T. Neugebauer

A recent Avery et al. fixed point theorem is applied to show the existence of a positive solution of the second order boundary value problem $x'' + f(x(t)) = 0,\ t \in (0,1)$ with boundary conditions $x(0) = x(1) = 0$. A corollary and examples are provided.

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*Solutions for Second Order Linear ODEs near Irregular Singular Points*

**Jeremy Mandelkern**
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Coauthor: Charles Fulton

We obtain a fundamental system of solutions near the irregular singular point $x = 0$ (having Poincare rank 1) of the Sturm-Liouville equation with strongly singular potential

$$-y'' + \frac{1}{x^4}y = \lambda y, \quad 0 < x < \infty,$$

and normalize them so as to satisfy the properties (i) $y(x, \overline{\lambda}) = \overline{y(x, \lambda)}$ and (ii) $y(x, \lambda)$ entire in $\lambda$ for each fixed x. At the crux of our new method are changes of variable that factor away the $\lambda$ - independent singular behaviour, and thereby afford solution for the $\lambda$ - dependent part of the solution by employing Volterra Integral Equations of the second kind. The solution technique represents an analog of the standard power series method for generating the solutions of the Fourier equation, $y'' + \lambda y = 0$, and we obtain convergent representations of the solutions.

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*Analysis of Steady States for Classes of Reaction-diffusion Equations with U-shaped Density Dependent Dispersal on the Boundary*

**Quinn Morris**
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Jerome Goddard II, Catherine Payne, Jordan Price, and R. Shivaji

We consider positive solutions to equations of the form

$$\begin{cases} -\Delta u = \lambda u(1-u), & x \in \Omega, \\ \frac{\partial u}{\partial \eta} + \gamma\sqrt{\lambda}(u-A)^2 u = 0, & x \in \partial\Omega, \end{cases} \tag{1}$$

where $\lambda > 0, \gamma > 0, A \in (0,1)$ are parameters, $\Omega$ is a bounded domain in $\mathbb{R}^n$; $n \geq 1$ with smooth boundary $\partial\Omega$ and $\frac{\partial u}{\partial \eta}$ is the outward normal derivative. Such models arise in the study of population dynamics in a habitat $\Omega$ when the population exhibits U-shaped density dependent dispersal on the boundary. We analyze the persistence of the population (existence, non-existence, uniqueness and multiplicity of positive solutions) as the patch size ($\lambda$) and the hostility of the outside matrix ($\gamma$) vary. We obtain results when $\Omega = (0,1)$ via a quadrature method, and when $\Omega$ is any bounded domain in $\mathbb{R}^n$; $n > 1$ by the method of sub-super solutions.

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*Time Sensitive Functionals in Real Time*

**Kizza M Nandyose**
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We study a class of monotone delayed marked point processes that model stochastic networks (under attacks), status of queueing systems during vacation modes, responses to cancer treatments (such as chemotherapy and radiation), hostile ambushes in economics and warfare. We are interested in the behavior of such a process about a fixed threshold. It presents an analytic challenge, because of the arbitrary nature of random marks. We target the first passage time, pre-first passage time, the status of the associated continuous time parameter process between these two epochs, and the status of the process upon these two epochs. A joint functional of these stochastic quantities is investigated in the transient mode. Analytically tractable formulas are obtained and demonstrated on special cases of marked Poisson processes.

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*Reaction-diffusion Equations with Non-differentiable Nonlinearities*

**Josef Navratil**
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Coauthor: Milan Kucera

The pattern formation in a system of two reacting and diffusing chemicals is a well known concept in mathematical biology. Such system is modelled by two reaction-diffusion equations. In this talk, the analysis of stationary states of more general system will be described. The system has a form

$$\begin{aligned} &u_t = d_1 \triangle u + b_{11} u + b_{12} v + n_1(u,v) \\ &v_t = d_2 \triangle v + b_{21} u + b_{22} v + n_2(u,v) + s_- v^- - s_+ v^+ \end{aligned} \quad \text{in } [0,\infty)\times\Omega, \qquad (2)$$
$$u = v = 0 \text{ on } \Gamma_D,\ \frac{\partial u}{\partial \vec{n}} = \frac{\partial v}{\partial \vec{n}} = 0 \text{ on } \Gamma_N,$$

where $\Omega$ is bounded Lipschitz domain, $\Gamma_D, \Gamma_N$ are disjoint complementary parts of its boundary, $d_1, d_2 > 0$ are diffusion coefficients considered as a bifurcation parameters, $v^-, v^+$ are the negative and positive part of $v$ respectively, $b_{ij} \in \mathbb{R}$ satisfy Turing conditions, $s_\pm \in L^\infty(\Omega, \mathbb{R})$ determine the strength of the unilateral source and sink and $n_1, n_2$ are small nonlinear perturbations. There is supposed that $(u,v) = (0,0)$ is a trivial solution of (2). The standard case is when $s_\pm \equiv 0$. It is well known that there exist two sets $D_U, D_S \subset \mathbb{R}^2_+$ of parameters and $\overline{D_U \cup D_S} = \mathbb{R}^2_+$. Stationary, but spatially non-homogeneous solutions of (2) bifurcate from the border between $D_U$ and $D_S$. In this talk, there will be described what happens when $\|s_+\|_\infty, \|s_-\|_\infty$ are positive, i.e. when there is a source and a sink. When there are mixed or Dirichlet boundary conditions and $\|s_\pm\|_\infty$ is sufficiently small, then there exist also bifurcation points in $D_S$, but still "close" to $D_U$. If the system has Neumann boundary conditions and sources are sufficiently large, then for any $d_1 > y_1$ there exists $d_2 > 0$ such that $(d_1, d_2) \in D_S$ is a critical point of (2). To get such results the variational and topological degree methods are combined and properties of generalized eigenvalues of positively homogeneous operators are studied.

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*Convolutions and Green's Functions for Fractional Boundary Value Problems*

**Jeffrey T. Neugebauer**
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Coauthor: Paul Eloe

We consider families of two-point boundary value problems for fractional differential equations where the fractional derivative is assumed to be the Riemann-Liouville derivative. The problems considered are such that appropriate differential operators commute and the problems can be constructed as nested boundary value problems for lower order fractional differential equations. The Green's functions are then constructed as convolutions of lower order Green's functions. Comparison theorems are known for the Green's functions for the lower order problems and so, we obtain analogous comparison theorems for the two families of higher order equations considered here. We also show by example that anticipated positivity of Green's functions can fail in the case when lower order fractional derivatives do no commute.

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*Sensitivity Analysis in Poro-Visco-Elastic Models*

**Marcella Noorman**
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Coauthors: H.T. Banks, Kidist Bekele-Maxwell, Lorena Bociu, and Giovanna Guidoboni

Poro-elastic and poro-visco-elastic models find many applications in bioengineering and medicine. Inspired from applications in geophysics and petroleum engineering, they are more and more frequently being applied to biological tissues. For many of these biological applications, the boundary data plays a crucial role. In a recent theoretical and numerical analysis of poro-elastic and poro-visco-elastic models, the time regularity of the imposed boundary traction was identified as a crucial factor in guaranteeing boundedness of the solutions. Here, we further extend that analysis by studying the sensitivity of model solutions to the imposed boundary traction. Nonlinear coupling in the model causes difficulty when implementing traditional methods for sensitivity analysis (such as sensitivity equations). For this reason, we use instead the lesser known complex step method which uses the Cauchy-Riemann equations to compute sensitivities.

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*Classification Schemes of Nonoscillartory Solutions for two Dimensional Time Scale Systems*

**Ozkan Ozturk**
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Asymptotic properties of solutions for nonlinear systems are significant in order to obtain enough information about the behavior of systems. We deal with a two dimensional time scale nonlinear system and show the (non)existence of nonoscillatory solutions by using most well known fixed point theorems. We also provide several examples, which validate our theoretical claims.

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*Approximation of Linear Neutral Delay Differential Equations*

**Catherine Payne**
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We consider semidiscrete approximation of a linear neutral delay differential equation of the form

$$\frac{d}{dt}\left[x(t)+\sum_{k=1}^{n} C_k x(t-r_k)\right] = Ax(t)+\sum_{k=1}^{n} B_k x(t-r_k)$$

with appropriate initial data. We assume that $A$, $B_1, B_2, \ldots, B_n$ and $C_1, C_2, \ldots, C_n$ are complex

$m \times m$ matrices. We reformulate the neutral equation as an abstract Cauchy problem $\dot{z}(t) = \mathcal{A}z(t)$ and discuss the construction of an approximation scheme which yields convergence for both the operator $\mathcal{A}$ and its adjoint. This property is needed in some control problems. Finally, we discuss some examples to compare this result with existing approximation schemes.

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*Early Host Pathogen Interaction in an Inhalational Anthrax Infection*

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Inhalational anthrax is a potentially fatal form of an anthrax infection which initiates after the inhaled spores are deposited in the lung, phagocytosed by immune cells, and subsequently transported to nearby lymph nodes. It is not clear if any deposited dose amount results in a fatal outcome and experimental work suggests that a threshold of deposited spores exists. To better understand the early disease dynamics of the host-pathogen interaction, we develop a mathematical model consisting of ordinary differential equations for spore and bacterial populations designed in accordance with an *in vitro* experimental protocol. The experimental data are used to estimate model parameters. Initial numerical results suggested the need to model two distinct subpopulations of anthrax bacteria: newly germinated bacteria and fully vegetative bacteria. Also, varying functional forms for germination and killing mechanisms suggest no clear dependence in intracellular burden of infected phagocytes. Additional modeling results suggest that intracellular germination and macrophage killing are more effective in the 1:20 scenario because the intracellular spore burden per macrophage as estimated by the model is closer to a 1:1 ratio than that of the actual 1:1 scenario.

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*Signal Flow Design of Fast DST Algorithms*

**Sirani M. Perera**
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Applications of FFT have spread into quite a diverse field of applied mathematics and engineering. It is sufficient to quote "FFT has changed the face of science and engineering so that it is not an exaggeration to say that life as we know it would be very difficult without FFT" by C. Van Loan to illustrate what an amazing accomplishment FFT has been. FFT is used to compute Discrete Fourier Transform (DFT) and its inverse efficiently. The Discrete Sine Transform (DST) is a Fourier-related transforms similar to the DFT, but using purely real matrix. Though there are eight versions, depending on applications in transform coding and digital filtering of signals, we consider DST matrices that vary from type I to type IV.

In this talk, we use a matrix factorization technique to introduce fast DST I-IV algorithms based on sparse, scaled orthogonal, and rotation/rotation-reflection matrices. Once the factorizations are established we analyze the arithmetic complexity and elaborate on numerical results. Finally, the language of signal flow graph representation of digital structures is used to describe these DST algorithms having $(n-1)$ points signal flow graph for DST-I and $n$ points signal flow graphs for DST II-IV for an arbitrary even integer $n$.

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*Identification of Parameters in Mathematical Biology*

**Roby Poteau**
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We consider the inverse problem for the identification of constant parameters for systems of nonlinear ODEs arising in mathematical biology. We implement a numerical method suggested in U.G.Abdulla, Journal of Optimization Theory and Applications, 85, 3(1995), 509-526. The method

combines Bellman's quasilinearization with sensitivity analysis and Tikhonov's regularization. We apply the method to various biological models such as the classical Lotka-Volterra system, bistable switch model in genetic regulatory networks, gene regulation and repressilator models from synthetic biology. The numerical results and application to real data demonstrate the quadratic convergence.

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*Qualitative Properties of Solutions to Nonlinear Parabolic PDEs with Double Degenerate Fast Diffusion*

**Adam Prinkey**
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We consider the problem of interface development and local behavior of solutions near the interface in the following Cauchy problem for the nonlinear double degenerate parabolic PDE with reaction and fast diffusion:

$$u_t = \Big(|(u^m)_x|^{p-1}(u^m)_x\Big)_x + bu^\beta,\ x \in \mathbb{R},\ t > 0;\ u(x,0) = C(-x)_+^\alpha$$

The problem arises in applications involving turbulant filtration of material through a porous media. The interface behavior is determined by the competition between the diffusion and the reaction terms. The full solution for the reaction-diffusion equation ($p = 1$) was given in 2002 [U. G. Abdulla, Nonlinear Analysis: Theory, Methods and Applications, 4, 2002, 541-560]. Our aim is to apply the methods of this paper to give a full classification for double degenerate reaction-diffusion equations with fast diffusion ($0 < mp < 1; m, p > 0$). Unlike in the case of slow diffusion ($mp > 1$), infinite speed of propagation of the interface is possible. First we apply the nonlinear scaling method to identify which term dominates in the various regions of the $(\alpha, \beta)$-parameter space. We then construct super/subsolutions and apply special comparison theorems in irregular domains to prove explicit formulae for the interface (where it exists) and local solution, with precise estimations up to constant coefficients.

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*Traveling Wave Solutions of R-D Systems with Application in Pattern Formation and Chemical Waves*

**Yuanwei Qi**
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Coauthors: Xinfu Chen, Zhi Zheng and Yajing Zhang

In this talk I shall report some recent progress on existence, multiplicity and stability of traveling waves of a class of reaction-diffusion systems which have direct application in biological pattern formation and chemical waves. This is a joint work with Xinfu Chen, Zhi Zheng and Yajing Zhang.

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*Stability In Nonlinear Delay Volterra Integro-differential Systems*

**Youssef N Raffoul**
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We employ Lyapunov functionals to the system of Volterra Integro-differential equations of the form

$$x'(t) = Px(t) - \int_{t-r}^{t} C(t,s)g(x(s))ds,$$

and obtain conditions for the stability of the of the zero solution. In addition, we will furnish an example as application.

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*Wave Propagation in a Noisy System near a Saddle Node on a Limit Cycle Bifurcation*

**Mahbubur Rahman**
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We develop and apply a method of stochastic approximation to a canonical model arises in mathematical neuroscience described by a parametric noise. We also investigates the role of noise in system.

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*Existence of Minimal and Maximal Solutions to Riemann-Liouville Fractional Integro−differential IVP*

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In this work we investigate integro−differential initial value problems with Riemann Liouville fractional derivatives where the forcing function is a sum of an increasing function and a decreasing function. We will apply the method of lower and upper solutions and develop two monotone iterative techniques by constructing two sequences that converge uniformly and monotonically to minimal and maximal solutions."

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*Distribution of Recurrent Outbreaks in a Stochastic Bi-trophic Ecosystem*

**Susmita Sadhu**
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We study the effect of stochasticity, in the form of Gaussian white noise, in a predator-prey model with two distinct time-scales. The prey is assumed to exhibit a faster dynamics than its predator leading to a system of slow-fast equations. We explore the effect of random perturbations in the excitable regime prior to the Hopf bifurcation. The stochastic model admits several kinds of noise driven mixed-mode oscillations that capture the intermediate dynamics between cycles of population outbreaks. We study the distribution of the random variable N, representing the number of small oscillations between successive spikes, as a function of the noise intensity and the distance to the Hopf bifurcation. Finally, if time permits, we will estimate the probability of repeated outbreaks by transforming the model into a suitable form.

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*Computing Reduced Order Models Using Stochastic Approach in Diffuse Optical Tomography*

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Coauthors: Eric de Sturler and Serkan Gugercin

Diffuse Optical Tomography (DOT) in medical image reconstruction presents huge computational challenges since we need to solve many large-scale discretized PDEs for each evaluation of the misfit or objective function. Moreover, in the nonlinear case each time the Jacobian is evaluated an additional set of systems must be solved. One can use reduced order models to reduce this cost. However, computing the reduced order model still requires solving many systems. In this talk, we discuss how computing reduced order models using stochastic approach can significantly reduce the cost of the inversion process in DOT.

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*Approximation of Solutions to the Mixed Problem on Lipschitz Domains*

**Morgan Schreffler**

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Coauthor: Russell Brown

The mixed problem, or Zaremba's problem, has many physical applications. In this talk we consider a method for approximating solutions to the mixed problem on a Lipschitz domain $\Omega$. Further, we propose a method for solving the approximate mixed problem using layer potentials.

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*On Stable Parameter Estimation for an ODE Constrained Optimization Problem in Epidemiology*

**Michael Sheppard**
*Georgia Tech*

Stable parameter estimation problems involving systems of non-linear differential equations arise frequently in the setting of epidemic models of infectious diseases and their transmission dynamics. Here we consider a classical SEIR compartmental model governed by a system of non-linear differential equations to model the most recent Ebola Virus Disease (EVD) outbreak of 2014. Stable and accurate estimations of the corresponding parameters can give crucial information to public health officials to implement successful and necessary intervention measures and programs. Moreover, these parameters can allow for disease forecasting and predicting significant turning points of an epidemic. Our compartmental model involves a time-dependent transmission rate between susceptible and exposed classes. Rather than assuming a known shape or form of the transmission rate determined through a finite set of parameters, the transmission rate is discretized as a linear combination of Legendre polynomials. The compartmental model at hand is reduced to a linear Volterra integral equation of the first kind. As with most parameter estimation problems in epidemiology, the inverse problem is highly ill-posed and unstable. To combat this instability, we suggest two approaches. First through Tikhonov regularization in the sense of linear least squares, and secondly by using a truncated singular value decomposition to solve the least squares problem at hand. Both approaches are supported with multiple numerical experiments involving recovery of the transmission rate and fitting to real incidence data.

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*A Model Reduction Algorithm that Preserves the Structure of Dynamical Systems with Nonlinear Frequency Dependence*

**Klajdi Sinani**
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Coauthors: Serkan Gugercin and Christopher A. Beattie

Very large-scale dynamical systems, even linear time-invariant systems, can present significant computational difficulties when used in numerical simulation. Model reduction is one response to this challenge but standard methods often are restricted to systems that are presented as standard first-order realizations; in the frequency domain such systems will be linear in the frequency parameter. We consider here dynamical systems with a nonlinear frequency dependence; systems for which either a standard first-order realization is unknown or inconvenient to obtain. Such systems may nonetheless have realizations that reflect important structural features of the model and we may wish to retain this structure in any reduced model used as a surrogate.

In this work, we present a structure-preserving model reduction algorithm for systems having quite general nonlinear frequency dependence.

We take advantage of recent algorithms that produce high quality rational interpolants to transfer functions that only require transfer function evaluation, thus allowing for nonstandard realizations that are nonlinear in the frequency parameter. However, our final reduced model will have a structure that reflects the structure of the original system, and indeed, may not have a rational transfer function. We illustrate our approach on a benchmark problem that offers a transcendental transfer function.

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*On Stable Reconstruction of Disease Parameters by Reduced Iteratively Regularized Gauss-Newton Algorithm*

**Alexandra Smirnova**

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Gerardo Chowell-Puente, Linda deCamp, Seyed Moghadas, and Michael Sheppard

For an emerging disease, the principal goal is to construct a reliable computational algorithm in order to quantify the most significant parameters describing the nature of impending epidemic. Another crucial task is to understand how soon after the emergence of a new disease, key parameters such as the epidemic size and the epidemic turning point can be projected.

We propose a regularized numerical method for early estimation of such parameters, and investigate its characteristics for convergence and computational stability. We apply this method to the generalized Richards model

$$\frac{dC}{dt} = rC^p \left[1 - \left(\frac{C}{K}\right)^a\right], \tag{3}$$

where $r$ is the intrinsic growth rate, $a$ measures the extent of deviation from the S-shaped dynamics of the classical logistic growth model, and $K$ represents the epidemic final size, defined as the total number of infections throughout the epidemic. When $p = 1$, (3) is known as the Richards model and has the analytical solution for the cumulative incidence, given by

$$C(t) = \frac{K}{\left[1 + ae^{-ar(t-\tau)}\right]^{1/a}}, \tag{4}$$

with $\tau$ being the inflection point of $C$. If $p \neq 1$, (3) has no closed form solution and must be solved numerically, although a solution may be expressed in the form of an infinite series. At the early stages of the epidemic, this model allows the capture of different growth profiles ranging from constant incidence ($p = 0$), polynomial (or sub-exponential) growth ($0 < p < 1$), to exponential growth ($p = 1$). The maximum incidence in the generalized Richards model is

$$C'(\tau) = \frac{raK^p}{p} \left(\frac{p}{a+p}\right)^{1+\frac{p}{a}}.$$

Estimation of the time ($\tau$) at which this maximum occurs for an emerging outbreak provides important information on the time-window available to implement the necessary intervention policies to reduce the number of infections.

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*Uniqueness Results for Classes of Semipositone p-Laplacian Problems*

**Byungjae Son**
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Coauthors: R. Shivaji and Inbo Sim

We consider steady state reaction diffusion equations on the exterior of a ball, namely, boundary value problems of the form :

$$\begin{cases} -\Delta_p u = \lambda K(|x|) f(u) & \text{in } \Omega_E, \\ Bu = 0 & \text{on } |x| = r_0, \\ u \to 0 & \text{when } |x| \to \infty, \end{cases} \tag{5}$$

where $\Delta_p z := \operatorname{div}(|\nabla z|^{p-2}\nabla z)$, $1 < p < n$, $\lambda > 0$, $\Omega_E := \{x \in \mathbb{R}^n \mid |x| > r_0 > 0\}$ and the boundary operator B is either $Bu \equiv u$ or $Bu \equiv \frac{\partial u}{\partial y} + c(u)u$ where $c \in C((0,\infty),(0,\infty))$ and $\frac{\partial u}{\partial \eta}$ is the outward normal derivative of $u$ on $|x| = r_0$. Here the weight function $K \in C^1([r_0,\infty),(0,\infty))$ satisfies $\lim_{r\to\infty} K(r) = 0$, and the reaction term $f \in C^1[0,\infty)$ is strictly increasing and satisfies $f(0) < 0$, $\lim_{s\to\infty} f(s) = \infty$, $\lim_{s\to\infty} \frac{f(s)}{s^{p-1}} = 0$ and $\frac{f(s)}{s^q}$ is nonincreasing on $[a,\infty)$ for some $a > 0$ and $q \in (0, p-1)$. We establish uniqueness results for positive radial solutions for $\lambda \gg 1$.

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*Boundary Value Problems for the Anisotropic Maxwell System*

**Eric Stachura**
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Boundary value problems for the time-dependent, anisotropic Maxwell system are analyzed in a bounded, Lipschitz domain in $\mathbb{R}^3$. The permittivity $\varepsilon$ and the permeability $\mu$ are parameters which determine the propagation of radiation in a material, and here are assumed to be $3 \times 3$ matrices depending on position. Motivated by an inverse problem in Electrical Impedance Tomography, solutions are obtained with general boundary data and under general assumptions on the material parameters.

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*Selected problems of non-linear dynamics*

**Alex Stokolos**
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Coauthors: D.Dmitrishyn and A.Khamitova

The problem of effective detecting and stabilizing of periodic solutions in non-linear autonomous discrete dynamical systems will be addressed.

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*Rate of Convergence of Solutions of Linear Volterra Difference Equations*

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Coauthor: Martin J. Bohner

We study the asymptotic behavior of solutions of a scalar linear Volterra sum-difference equation. A positive lower bound for the rate of convergence of asymptotically stable solutions is found by assuming the kernel is a positive and summable sequence such that $k(t+1)/k(t) \to 1$ as $t \to \infty$.

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*Interactions of Different Shock Waves and Expansion Waves*

**Ligang Sun**
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In this talk, I will introduce the dominant equations normal shock waves and oblique shock waves, including the Rankine-Hugoniot equation and Prandtl's velocity equation in these two kinds of shock waves. Then the interaction principals of shock waves will be discussed. After the mathematical analysis for the different interactions of shock waves and expansion waves, several simulations of different combinations of shock waves and expansion waves will be presented to show specific phenomenon and properties.

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*Existence of Discontinuous Local Minimizers to a Class of Semilinear Integral Equations*

**Jeremy Trageser**
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In this work we study a nonlocal bistable equation which arises as the Euler-Langrange equation of a nonlocal van der Waals type functional. We compare nonlocal interactions given by Green's functions of second-order differential equations and investigate stationary points with multiple discontinuous interfaces. We consider criteria under which the function does not admit a stable stationary point for a finite number of discontinuities. We also provide criteria which ensures the existence of a stable stationary point.

*Stability Results for Set Difference Equations*

**Gnana Bhaskar Tenali**
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Set Difference Equations (SDEs) occur in the description of observed evolution phenomena, since most measurements of time evolving variables are discrete. They also arise in the discretization methods for Set Differential Equations. In this talk, we present some earlier results on the stability of Set Difference Equations. A few recent results on the stability in terms of two measures and Lyapunov stability of fixed points for discrete dynamical systems in $K_c(\mathbb{R}^n)$, by Slyn'ko and others are also discussed.

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*Study of the Chaotic Vibration of High Dimensional PDEs*

**Jing Tian**
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Coauthor: Bingsheng Zhang

In this talk, I will mainly discuss the chaotic oscillation of the two-dimensional non-strictly hyperbolic equations due to an energy-injection boundary condition and a distributed self-regulation boundary condition. Numerical simulations will be provided.

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*Shear Bandings of a Thixotropic Fluid Model*

**Taige Wang**
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The PEC (partially extending strand convection) model is able to describe thixotropic yield stress behavior in the limit where the relaxation time is large relative to the retardation time. In this paper, we discuss the development of shear bands in a Poiseuille flow which is started up from rest with an imposed pressure gradient. We analyze the asymptotic limit of large relaxation time; the small parameter$\epsilon$ measures the ratio of retardation time to relaxation time. We determine the position and width of shear bands as a function of time. We identify an initial phase of "fast yielding" during which the width of the transition between high and low shear rate regions behaves like $t^{-3}$. This continues until t (measured on the scale of the retardation time) is on the order of $t^{-1/3}$. Then there is a phase of "delayed yielding" during which the width of the transition is of order $\epsilon$. Eventually, the width sharpens as $1/(\epsilon^2 t^3)$. We also show how these results are modified by introducing Korteweg stresses which prevent the transition from becoming infinitely sharp and also change the location where the transition takes place.

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*Persistence criteria for the nonlocal niche model and applications*

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Long range dispersal is a common phenomenon in biology and ecology. To have a better understanding of the evolution of biodiversity in some ecosystem, there is a need to understand the influence of nonlocal dispersals on the survival/persistence of a population.

In this talk, I will report on a recent study concerning persistence criteria in some nonlocal models on temporal and spatial heterogeneous environment.

I will first present some spectral theory of the associated eigenvalue problem, such as the existence of the principal eigenvalue, and the asymptotic behaviors of the generalized principal eigenvalue with respect to its underlying parameters. As a consequence, I will discuss the applications of these results to the evolutionary invasion analysis. Secondly, I will show some results of the eigenvalue problem with indefinite weight functions, which have practical importance in the context of reserve design or pest control.

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*On Nonoscillatory Solutions of Four-dimensional Time-scale Systems*

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There are a number of cases in which the theory of time scales can serve as a model in an abundance of applications. Hence, studying the properties of nonoscillatory solutions of four-dimensional time-scale systems so as to unify these discrete and continuous cases is of great importance. In this study, the main aim is to classify nonoscillatory solutions of four-dimensional time-scale systems of first order dynamic equations on time scales.

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*Landau Damping in Pseudo-Relativistic Plasmas*

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We examine the phenomenon of Landau Damping in (pseudo)relativistic plasmas via a study of the relativistic Vlasov-Poisson system (rVP) on the torus for initial data sufficiently close to a spatially uniform steady state. We find that if the steady state is regular enough and if the deviation of the initial data from this steady state is small enough in a certain norm, the evolution of the system is such that its spatial density approaches a uniform constant value quasi-exponentially fast (i.e. like $\exp(-C|t|^{\overline{\nu}})$ for $\overline{\nu} \in (0,1)$). We note that none of our results require spherical symmetry (a crucial assumption for many current results on rVP).

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*On Energy-stable Schemes for Complex-fluid Hydrodynamic Equations*

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Complex fluids are fluids whose micro-structure have impact on the fluid macroscopic properties, which include complex fluid mixtures of different types. In this talk, I will first present a systematic development of a general hydrodynamic model for complex fluid system using the generalized Onsager relation. Then, a semi-discrete scheme to solve this general model, which satisfies the discrete energy dissipation law, will be presented. Specific tricks on linearizing and decoupling the schemes will be presented for particular reduced models. In the end, several 3D simulations will be shown to illustrate the effectiveness of our schemes.

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*Attractors of Nonlinear Population Balance Equations with Feedback Control*

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In this talk, we consider a class of quasilinear partial differential equations corresponding to conservation laws with one space variable. We represent these equations as an abstract differential equation in a suitable Hilbert space. This abstract differential equation admits an equilibrium which is not asymptotically stable in the general case. To stabilize the equilibrium, we apply a finite-dimensional control in the right hand side of the abstract differential equation. We use a quadratic Lyapunov functional to construct a bounded feedback law provided that the state of the system is fully observable. Then the limit behavior of the trajectories is analyzed by means of LaSalle's invariance principle. To estimate the attractor of the closed-loop system, we apply compactness theorems from [Zuyev A., Partial stabilization and control of distributed parameter systems with elastic elements, Springer, 2015].

In particular, it is shown that the equilibrium of the closed-loop system is asymptotically stable under additional assumptions concerning the Fourier transform of the control coefficients.

These results are applied to nonlinear population balance equations that represent the dynamics of crystallization and granulation processes.