

## Crystalline and Complex Solutions to the Vector Allen-Cahn Equation.

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We study the problem of finding bounded entire solutions  $u: \mathcal{R}^n \rightarrow \mathcal{R}^m$  to the vector Allen-Cahn equation  $\Delta u - W_u(u) = 0, x \in \mathcal{R}^n$  where  $W: \mathcal{R}^m \rightarrow \mathcal{R}$  is a smooth, symmetric, nonnegative multi-well potential that vanishes at  $N$  distinct points  $a_1, a_2, \dots, a_N \in \mathcal{R}^m$ , and  $W_u = \Delta W$ . Even in the case  $m = 1$  and  $N = 2$ , the problem of identifying all solutions is very difficult (see the work stemming from De Giorgi's conjecture, and relatively recent work by Dancer, Malchiodi, Del Pino - Kowalczyk - Wei, for instance).

Here we present a large family of symmetric solutions in the general case with  $W$  described above. This is joint work with Giorgio Fusco and Panayotis Smyrnelis.